Name: Key

Math 221: Test 2 - 6/15/15

Do all work on the test below. You must show all work to receive full credit.

1. Determine which model is being described in each problem below, and briefly explain your answers. You do not need to solve the actual word problem. [5 Points Each]

- Set Model
- Number Line Addition
- Take Away
- Missing Addition
- Comparison
- Number Line Subtraction
- Repeated Addition
- Number Line Multiplication
- Array
- Area
- Cartesian Product
- Partition
- Measurement

(a) You've just gotten a new job of cleaning houses. This morning, your company was given 15 requests to have houses cleaned. You were assigned to clean 3. How many do they have left to give to others?

Take-Away: 3 houses were removed from their list of 12.

(b) If you make \$15 an hour while cleaning and work for 9 hours, how much money do you make?

Number Line x: \$15 → \$15 → \$15 → ... → \$15 (continuous flow)

(c) In one particular house, suppose you pick up 12 action figures in the boy's room and 7 dolls in the girl's room. How many toys did you pick up altogether?

Set Model: Action figures and dolls are two disjoint sets.

(d) After cleaning a refrigerator, you put 27 items back onto the 3 shelves evenly. How many items did you put on each shelf?

Partition: Given number of groups and need to know amount in each group.

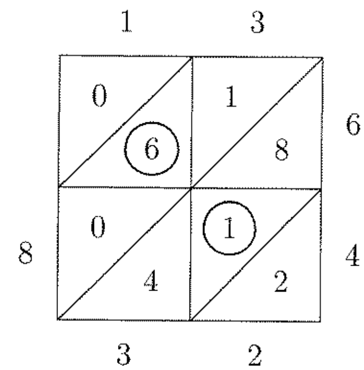
2. Consider the set $\{2, 4, 6, 8, ...\}$ under multiplication. Circle the properties that hold under this situation, then explain your answers for Closure and Identity. [8 Points]

(Closure) (Associative) (Commutative) Identity

Multiplying two even numbers returns an even number

$1 \notin \{2,4,6,8\}$, and no value can be used that returns the other value when multiplied

3. Explain how the lattice method of multiplication works in the following problem by answering the following questions. (Note: All numbers are in base ten.) [5 Points Each]



(a) Why does the circled 6 get added into the hundreds place?

We actually did $10 \times 60 = 600$, so the 6 is in the hundreds place.

(b) Why does the circled 1 get added into the tens place?

When we do $3 \times 4 = 12$, we regroup the 10 units into 1 ten.

4. Using the problem 432 − 125 (in base ten), explain what is actually going on in the regrouping step. Base ten blocks are wise to consider here. [5 Points]

$$\begin{array}{r} 4\,\overset{2}{\cancel{3}}\,\overset{12}{\cancel{2}} \\ -125 \\ \hline \end{array}$$

To subtract 5 units from 2 units, we regroup one of our tens into 10 units. Now we can subtract 5 from 12.

5. Use the specified algorithm to solve the given problem. [8 Points Each]

(a) $1010_{five} \div 14_{five}$ (Long Division Algorithm)

24_{five} R 4_{five}

14) 1010
- 33
1 2 10
-121
4

1	2	3
14	14	14
× 2	× 3	× 4
33	102	121

(b) $134_{five} \times 42_{five}$ (Standard Algorithm)

$$\begin{array}{r} 134 \\ \times\ 42 \\ \hline 323 \\ +12010 \\ \hline 12333_{five} \end{array}$$

(c) $4203_{\text{five}} - 2414_{\text{five}}$ (Equal Additions Algorithm)

$4203 \xrightarrow{+1} 4204 \xrightarrow{+30} 4234$

$-2414 \xrightarrow{+1} -2420 \xrightarrow{+30} -3000$

1234_{five}

$2414 \xrightarrow{+1} 2420 \xrightarrow{+30} 3000$

6. Determine whether 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 divide 1980. Briefly justify each one. [13 Points]

(2) 2|0, so 2|1980

(3) 1+9+8=18

3|18, so 3|1980

(4) 4|80, so 4|1980

(5) ends in 0, so 5|1980

(6) 2,3|1980, so 6|1980

(7) $198 - 2 \cdot 0 = 198$

$19 - 2 \cdot 8 = 3$

7 ∤ 3, so 7 ∤ 1980

(8) 8 ∤ 980, so 8 ∤ 1980

(9) 9|18, so 9|1980

(10) ends in 0, so 10|1980

(11) 1−9+8−0=0

11|0, so 11|1980

2, 3, 4, 5, 6, 9, 10, 11

7. Determine if 221 is prime or composite. You do not have to justify your divisibility tests, but write down the result of each value that you check. [6 Points]

$221 = 13 \cdot 17$ $\qquad \sqrt{221} \approx 14.87$

Composite

8. Find the greatest common divisor of 64 and 48 using the Intersection of Sets method (the method in which you write all divisors and look for common ones). [6 Points]

$D_{64} = \{1, 2, 4, 8, 16, 32, 64\}$

$D_{48} = \{1, 2, 3, 4, 6, 8, 12, 16, 24, 48\}$

$D_{64} \cap D_{48} = \{1, 2, 4, 8, 16\}$

$GCD(64,48) = \boxed{16}$

9. Find the least common multiple of 42 and 120 using the Prime Factorization method (the method in which you compare prime factorizations). [8 Points]

$42 = 2 \cdot 3 \cdot 7$

$120 = 2^3 \cdot 3 \cdot 5$

$LCM(42,120) = 2^3 \cdot 3 \cdot 5 \cdot 7 = 840$

The following bonus questions should not be attempted until you have solved every other question and checked your answers. Please see me if you need extra paper.

Bonus 1: Compute $463_{\text{eight}} \times 75_{\text{eight}}$ using any method of your choice. [5 Points]

Bonus 2: Write a believable divisibility test for 24 by trying something similar to the divisibility test for 26. Show that 5016 and 5568 is divisible by 24 using your test. [5 Points]

1)

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   463
 ×  75
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  2777
+41450
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144447 eight
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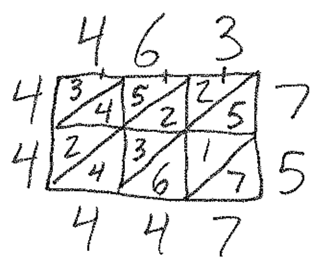

2) $24|n$ if and only if $3, 8|n$.

③ $5+0+1+6=12$

$3|12$, so $3|5016$

⑧ $8|16$, so $8|5016$ $\Rightarrow 24|5016$

③ $5+5+6+8=24$

$3|24$, so $3|5568$

⑧ $5|568$, so $5|5568$ $\Rightarrow 24|5568$